

CHAOS AND COMPLEXITY

Mathematics is many things besides modelling and simulation, but it has provided the most successful tools for it so far, and I want to talk about the application of these tools in the realm of complexity. In 1961, somebody whose name nobody knows invented floating point numbers, popularly called floats, and scientific computation, as it is known today, came into being. During the 1960s, a major innovation in technology called scientific computation took place. Today, everybody takes it so much for granted that you wouldn't even mention it in a conference on technical innovation. Then in the 1970s, we had chaos; it became as much of a household word as Einstein. The development and particularly the popularization of chaos theory was a direct consequence of floats and of the revolution of scientific computation that preceded it.

Floats were invented in 1961, and by 1962, people had seen chaos in digital computation. A popular book on this subject came out only recently, which shows that there is a certain lag between the scientific revolution and the popular understanding of it. In the 1980s, we were all talking about complexity, and there is a popular book on complexity also. It's called "The Dreams of Reason." Ironically, the popular book on complexity was published at the same time as the popular book on chaos, because the chaos book is by a journalist, but "Dreams of Reason" is by a scientist whose name is Heintz Pagels. Another book he wrote earlier is called "The Cosmic Code". Over and above advancing the frontiers of science, he was seriously interested in the most rapid possible communication of scientific progress to a popular audience, which seems a valuable activity to me. He was a physicist at Rockefeller University and president of the New York Academy of Science. He died recently in a mountaineering accident, and I want to dedicate this talk to Heintz Pagel.

Now, chaos might be a revolution; complexity, I think, is more like the gradual ascent of Mt. Everest, or maybe of some endless, infinite mountain chain. We have before us a series of increasingly complex systems that we live and work with. One example is so-called global modelling. In this area, scientists from various physical and earth sciences like physical oceanography, and plate tectonics, atmospheric physics, atmospheric chemistry, biogeography and so on, make individual models in their individual departments. They run them on their individual computers, and then they meet together in international conferences trying to figure out a so-called wiring diagram. Eventually the tectonics, the ocean, the atmosphere and all the gas exchanges, heat exchanges, energy exchanges and coupling connections between them are argued out, and an actual formula as to what's connected, how it's connected, how strong it is, etc., begins to emerge. These individual models for organisms of the whole system can then be plugged in together to run on a network of distributed supercomputers that try to simulate the long-run

behavior of a complex system.

This is primarily a physical system; a more complicated example might be the endocrine system of a mammal with some thousands of transmitter hormones that are connecting different organs through circulatory systems. Each organ has a department in a university with its models and its own computers, and eventually one can try to hook these all together. There is a fairly large effort at Los Alamos on the Cray Supercomputer to imitate the endocrine system of a mammal. It seems to me that this system is much more complex than the other because of the biochemistry and other aspects of the life systems involved. One of these systems, for example, called a stress response, involves the hypothalamus, the pituitary and the adrenal cortex. Upon a signal, the hypothalamus releases a transmitter that affects the pituitary, which releases another transmitter that causes the adrenal cortex to release cortisol, which makes the heart beat faster. This entire process could be set off by a barking dog, or a fire engine siren.

If you take the history of the social sciences one at a time and put them all together, you have history. Robert Artigiani is a pioneer in the development of the understanding of this system as a complex system. But it was already a very complicated system even before the Renaissance, because there were many nations. Nations are born, grow to maturity, fade away and die more or less like individual mammals. There are so many of them, and this makes the wiring diagram extremely complicated because of all the different things they exchange with each other, particularly verbal information like insults, threats, jokes, lies and -- what do you call it? -- espionage and the like. In my view this would be a much more complicated system than this one. There is kind of a downwards increasing scale of complexity.

Although this list of systems may be endless, I'd like to end it here artificially by defining one and calling it the whole thing. Under history we have all of the social sciences, so here is where we would put the management, for example, of a small organization, or of a government, or of a league of nations. Decisions in businesses, futurism, all these things can be included in history if we define history as something that deals with the past, the present, and also the future. All these systems are too complicated to study with the traditional mathematical models that were made popular by mathematical physics. Physical systems are by definition very simple systems. In order to understand such complex systems through the traditional strategy of modelling and simulation, we need to use the new technology of modelling and simulation, because classical analysis of mathematical physics will not do. On each level of complexity there are critical cases, systems that we must understand in order for our species and for the planet to survive as a living system. We need to have a better understanding of complex systems on that scale, particularly of the so-called world problematique which exists on this level, including the challenges of ecology.

If we don't succeed to increase our intelligence and our capability to understand complex systems through modelling and simulation, we won't survive. For its potential use in these practical problems I want to propose CDS - complex dynamical systems theory. This is a branch of mathematics that combines dynamical systems theory, a recently evolved branch of topology that is sometimes called dynamical topology; it includes catastrophe theory, chaos theory, bifurcation theory, practical geometry, and so on. Together with SD

(system dynamics), it is a branch of general systems theory, which is sort of a mathematical or differential equations branch. This reunion is a bit of a sociological problem, because dynamical system theory has evolved in the academic context, while general systems theory was rejected by academic science soon after its birth in World War II and has evolved as a separate discipline outside the academy. It's like matter and anti-matter. Academic science has so many branches: physics, chemistry, management, sociology etc. In general systems theory you have exactly the same branches, but all of them are totally disjointed. Academic mathematics contains dynamical systems theory, and general systems theory contains systems dynamics.

The dynamical systems theory is aided by academic mathematics, including topology. It has developed a detailed geometry and statistics of chaotic attractors and bifurcations. Meanwhile the general systems community developed to a high art the strategy of passing (or parceling?) an organization of global systems into subsystems based on certain arcane principles. By reuniting the matter and the anti-matter we end up with strategy CDS, which is a strategy for making successful models for complex systems. What the dynamical systems theory provides is the understanding of these things which happen necessarily in the complex systems. Dynamical systems have been studied by various people, but when you connect them up, without fail they're chaotic and have bifurcations. Without this theory you can understand only the simplest possible ones. What emerges by putting these two things together is something much greater than the sum of the parts, which is an incredibly powerful technique for modelling and simulating complex systems. So far, it has been used little, except on this level of complexity where they're just now trying to wire up individual dynamical systems into a network. Perhaps one reason for it is that the chaos revolution has taken place.

The main idea of chaos theory is that when you see chaotic data, you do not have to lay down and die. It's perfectly o.k. because we now have the technology for it. Here are the basic ideas of complex dynamical systems. (writing on board). I'll take a simple one. It'll have subsystems and connections. You have to understand the diagrams, you must understand the connections, and you need to understand what goes in the boxes. What goes in the boxes is the hardest thing to understand. They're called response diagrams. To explain them I wrote the four volumes called "Dynamics, the Geometry of Behavior". They present a particular view of a dynamical scheme.

A dynamical scheme is a dynamical system with parameters. Controlled parameters. The dynamical system is a system of first order, ordinary differential equations, autonomous and so on, but of several dimensions and such: $X_1 \dot{=} f_1$. X_1 up to X_n . (writing on board). But with parameters that would mean that you add in here somewhere some control parameter, and when you change that parameter, this function changes to a different one. A dynamical system has its own picture which is called a phase portrait. Or just a portrait. The portrait contains attractors, basins, and separatrices. These systems can take place in spaces of several dimensions but none that are necessarily flat. They could be a sphere, a torus or something like that, but I'll just draw a flat one.

These equations can be regarded as defining a velocity at every point. The rules of the game - called the dynamical rule - are these velocities. At each

point there is only one velocity. Your assignment, if you choose to accept it and play this game, is that you start anywhere, and after that, you have to follow the rules of the game, which are that whatever is the speed you're supposed to have there, you run at that speed. Then you get to a new point. There you have a new instruction, a new rule. You have to run at that speed. So you keep following the rules, taking one small step at a time, and then you move in a trajectory - that's playing the game.

Now it happens almost always that you end up by sort of slowing down and stopping somewhere. That is, if you put players all over the field and let them go, they'll eventually end up in a heap. That's called the attractor. So, one way trajectories can approach attractors is sort of straight in. That's called a nodal attractor. Another trajectory comes in here. Other ones come in here. That point is an attractor. Pink is dynamical rule, blue is called the trajectory and in these applications, one generally does not care about the trajectory. If you're shooting a basketball at the basket then it's the trajectory that you care about. But if you're wondering what will be the situation of the endocrine system after things settle down, then you care only about the attractors.

One hypothesis or axiom of this entire approach is that attractors are emphasized. One is trying to model the long run behavior of the dynamical system. This entire situation then, the complete portrait, has in it in this particular case only one attractor. That's called a monostable system. But even so, there are three different types of attractors which could take place there. This one is a point. That's called a static attractor. Then there can be a cycle, also called limit cycle. I call that a periodic attractor. And then there's some other kind which is a chaotic attractor.

Rather than go into detail about the technical characteristics of these three different kinds of attractors, I'd like to now connect them all together. The important thing is the attractor, and that's more important than its characteristics in most applications, so let's suppose that the chaotic attractor is the only one. This picture shows a very small chaotic attractor or attractors seen from afar - as a star or galaxy. In that case, ignoring the complication about the different types of attractors, I can draw a picture in which there are two attractors, which models a bistable portrait.

So here's another one. A little cloud. It also attracts all nearby players who play this game. But the ones that start on the left hand end of the field would end up at the left, and starting on the right half end of the playing field ends up in the right goal. So if this was, let's say, life, and this one death, if you play this game you'd like to know where to start out to end up with a desired result. So all the initial positions that places players who end up in this state is called isbasin (or his basin?) we try to color them in. Ok. So this one ends up there, I'll color that yellow. This one ends up there, in fact everything along here ends- so it may be some entire region like this ends up there, some other region like this - they all end up here and then there's a boundary in between which in practice it might be very important to know, which is called the separatrix. Ok? So now you know the attractors in this picture there are two. The basins, in this picture there are two, and the separatrices in this picture there's one. That's what you might have to know now. In many applications the two basins are intermingled and mixed as if you'd take this picture with the yellow molasses and the green

taffy and you stir it up like this and then you get this marbled effect. In practice that's the usual situation.

Now you know all about phase portraits. It is a good way to look at dynamical systems if you're only interested in the long run behavior, and therefore we could go back to this dynamical scheme which is a dynamical system with parameters, and that means that for every value of parameters you have another one of these pictures. When you change the parameter, this picture is seen to move about and what might move about is that the attractors might move, the basins might move, and the separatrix might move. Let's suppose that the parameter is one-dimensional. I'm going to take this picture, put it up here, and embed it in a family of so many more pictures in which the attractors, the separatrix, the basins are going to move. Ok? So let's put this one on this end - life, death, and definition of good choices for players wishing to live, and now we'll take the control parameter and move it this way. A lot of different things can happen. The simplest thing, and this is actually good enough for a lot of different applications - is that these three features, these two and this one, just move around as the control is changed. That's enough for neural nets (?) for example, systems that can recognize faces.

But what is more interesting and happens a lot -- we like to use these in applications to model sudden transitions of a system -- is bifurcation. One application is that, while moving along, this attractor moves closer and closer to its separatrix, going to the edge of its basin. Well, maybe the basin hardly moves at all, or moves this way only a little bit, and eventually it collides. And then the attractor, the basin, the separatrix of this state all vanish into the blue, leaving nothing but this one, which is more or less unchanged by the process. That means that over here we have two states, bistable system: on/off. Over here, only one state: off. And where these meet and mutually annihilate, that's called the bifurcation. And that will happen for the particular value of a control parameter. So if you're on, if the system is on and you take the control lever in your hand and move it that way, then essentially you turn it off. And you say, well, I don't know if I wanted that, I'll move it back again, and then it stays here. It stays off. So much insight about bifurcations of this type, which are called catastrophic bifurcations, is provided by the study of catastrophe theories. A theory which is extremely important.

If you want to understand this and use it in your work, you could not do better than by beginning with catastrophe theory. Interesting and useful pedagogic applications have been worked out in detail by people who care about these things. Unfortunately, the famous catastrophe theory of the late '60s and early '70s fell into ill repute because of a sociological and historical accident, so that few people have heard about it or are studying it now. That theory shows that all of the attractors involved are point attractors. The same thing happens to periodic attractors and to chaotic attractors. Point attractors disappear into the blue. That's a classical catastrophe. Periodic attractors also disappear into the blue. That's called a periodic fold. Chaotic attractors disappear into the blue as well. That's called a chaotic catastrophe, or chaotic fold. One of the really good introductions to this subject is Rosen's book, "Dynamical Systems and Biology", which was one of the first places where in the applied mathematical or theoretical biology (end of tape)

...(tape started in middle of sentence) especially rich when the number of controls is more than one and what happens if you have the speed and the amplitude of an oscillation. Have two control models(?). You can practice with classical catastrophe theory and then you can imagine that these -- I call them the loci of attraction -- are actually periodic or chaotic attractors moving around, appearing or disappearing.

There are three classes of bifurcation. (writing on board). One is a catastrophic bifurcation. Another is explosions, and a third is subtle bifurcations. So now we've seen one example of catastrophic bifurcation in this very simplest presentation. I'd like to give the simplest known examples of the other two also. By the way, this figure here is a response diagram. (writing on board). And so at that point there I completed a sentence that began here. So the response diagram is the basic unit or model of subsystem which goes with these boxes. If that one is like photographed on a two dimensional card and then you put it in here, that's what I mean goes in these things. So we have here two privileged dimensions, the control dimension which might have several actual dimensions, corresponding to the number of controls, and the dimension of these vertical cards, in this instance two, which is called the state dimension. Or the number of state variables. So if I add to this picture control and state system number one, and control and state system number two, then the coupling that goes in between I expressed informally as a - just directed line segment. It is actually a mathematical function which assigns to the states of one system controls of the next.

So now you know everything about complex dynamical system. Formal definition, everything is there. You make gigantic networks of things like this. Inside each box at a node goes a dynamical scheme. A dynamical system with parameters visualized as a response diagram gives another response diagram. Where do you get them? You have to go to scientists and ask them. They spent their lifetime working out models for the subsystems, and other scientists have conjectures as to connecting them up which can be thought about as a first approximation of what you're really going to do.

In the new connectionist paradigm of complex dynamical systems theory and neural nets, it is thought that the intelligence of a complex system is primarily in the connections rather than the sophistication of the nodes. By changing the connections only, you can retrain the model to behave qualitatively like the target system, even if the models and the nodes coming from the specialized scientists are all wrong. The Newtonian model is replaced by the Einsteinian model. That's very important in one branch of science, but when you connect up the whole thing, it might not matter. This may be wrong, but anyway, it's a starting hypothesis for building models for complex systems.

There are many complex systems. If you study, for example, the hypothalamus, the pituitary and the adrenal cortex, you will find that the hypothalamus and pituitary were very well known, but not the adrenal cortex. So you place the adrenal cortex in the blank box and put any system you want in it, and you can still get the model sort of to run. Once it is running, you can start replacing the model in the blank box X with different models derived from the Russian literature, or South American literature and so on, and if the behavior gets better, you feel that you are discovering through the technology of modelling and simulation the actual model for the blank box. Since a lot

of subsystems can be studied without destroying them, this might, in fact, be an important technique. I wish I could say that catastrophe theory is a huge theory that you can go out and learn from a text book, enabling you to perform miracles. But as a matter of fact, it is not a well developed antique branch of mathematics. It is a new branch in the process of being born. Its early development is taking place in co-evolution with its applications in the various sciences.

Let me conclude with a description of a kind of program for applying these methods in the world of science. Their increased understanding might actually affect our future. In the management of small systems, applications are well underway. The management sciences use general systems theory, dynamical systems, bifurcations, chaos, and so on. In the biological and physical sciences as well, applications are already in use. In many cases, the people who are carrying out these applications are unfunded and sometimes unemployed, because there is a certain resistance in the establishment to these new methods. In order to have a future, we have to address the problem in the sociology of science and technology. We have to do this in our own bailiwick, wherever we may be living and working. We have to think locally and act globally. We need the further development of the complex dynamical system branch of mathematics, and we need to actively encourage the total revolution of each and every branch of science so that it begins to make full use of the techniques already available to transcend its frontiers. In most cases this requires a better understanding of complex systems than is generally available now. If you're in a funding agency, you have to watch what you're funding, and if you're working in the lab, you have to be willing to risk your funding to continue your work.

If we care about the outcome of the human venture on this planet, we must act in a certain religious way with regard to this revolution of the sciences. It will not happen without the conscious effort to link our highest understanding to our future evolution. Our best hope lies in the unification of the sciences. We have to create, propose, fund, sponsor, and participate in conferences that are interdisciplinary, in order to make sure that we create compatible models in the different specialities which can be plugged in together in a distributed net of massive computers. Once we have done that, we have to go ahead and do it with whatever resources are available. It would make partnerships of at least two specialities possible -- sort of sex and marriage in the world of science. An evolution in the direction of the global synthesis of the sciences would then begin to take place, enabled by the availability, for the first time, of a strategy for modelling and simulation - a universal strategy such as the classical analysis in mathematics provided in the early days of the physical sciences, in physics, chemistry, electromagnetism, mechanics and so on. Let us imagine that this exercise could actually take effect in society, so that decisions made by individuals, organizations, governments and so on would be guided by the understanding provided by the model of the whole thing. This would increase the probability of the survival of the human species on planet Earth, if it is done in harmony with the ecology of the planet. An interesting possibility would be to inject this technology into the school system at the lowest level, in pre-kindergarten, kindergarten, and in the elementary grades. I'm thinking of some sort of colored NIX machines or MACIIs that are simplified models of the whole thing. This would give our children the capability for understanding complex systems intuitively, and the significance of local decisions on global

life would be obvious to them.

Since the problem facing the world is fairly immediate and the development of the social sciences around mathematical modelling and simulation is going to take quite a while, even if people were working on it, which, since there's no funding, they're not, we would like to find a shortcut so that it might more or less begin immediately without waiting for the full model to evolve. Maybe even the conception of a full model, and the fact that people in different places are working toward it, could in itself already achieve the fundamental goal. People get McArthur awards and Nobel prizes for small achievements, but any interdisciplinary work is punished in the institution. The function of the institution is to preserve the territory of the specialities. A planetary and peaceful society in harmony with the biosphere may be made possible by the creation of interdisciplinary networks of complex dynamical systems. Maybe science isn't the most important thing, but within science, including the science of management decision, futurism and so on, it would be the best that we could do. There would be parallel development on all levels if the education of the whole thing began in the earliest school grades. A certain amount of software suitable for this purpose already exists. For example, in the game "Balance of Power" that runs on the Macintosh, you can simulate a social game between nations. By and large, however, there is no reward in writing educational software. The state of hardware is in advance of what we need to begin this program, while the software is practically nonexistent. So that's the situation as I see it of complex dynamical system theory in the context of our actual lives and of society and the future.